

Lattice QCD computation of the SU(3) String Tension critical curve

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We investigate the critical curve of the string tension $\sigma(T)$ as a function of temperature in quenched gauge invariant SU(3) lattice gauge theory. We extract $\sigma(T)$ from the colour averaged free energy of a static quark-antiquark pair. To compute the free energy, we utilize a pair of gauge invariant Polyakov loop and antiloop correlations, and apply the multi-hit procedure to enhance the signal to noise ratio. We find that the string tension departs from the zero temperature σ_0 at $T \simeq 0.5T_c$. We cover the relevant temperature range from $0.5T_c$ up to the confinement temperature T_c using 54 different sets of pure gauge lattice configurations with four temporal extensions (4,6,8,12), different β and a spatial volume of 48^3 in lattice units.

I. INTRODUCTION

The string tension $\sigma(T)$ is a relevant order parameter to study confinement. While above the deconfinement temperature T_c the simplest order parameter is the Polyakov loop, below T_c the expectation value of a single Polyakov loop vanishes. Below T_c , to study the decrease of confinement with T a new order parameter must be used, and we utilize here the string tension. The string tension $\sigma(t)$ parameterizes the confining sector of the quark-antiquark potential, which increases linearly with distance. At finite temperature the quark-antiquark potential $V(r)$ is extended to the quark-antiquark free energy $F(r, T)$.

Moreover we are interested in the string tension and in the confining quark-antiquark free energy, since it leads to chiral symmetry breaking [1]. It also dominates the hadron spectrum. To study chiral symmetry and the hadron spectrum at finite T , It is important to know the string tension $\sigma(T)$ behaviour at all temperatures.

The existing lattice QCD results for the string tension critical curve have been computed by the Bielefeld group [2], who have studied in detail the heavy quark potentials in the confined and deconfined phases at finite temperature. In the confined phase, the Bielefeld Group presented results for the string tension in the region $[0.8T_c, T_c]$ with lattice size of $32^3 \times 4$ generated with a tree level Symanzik-improved gauge action. Their results confirmed a first order deconfinement phase transition, as expected for SU(3). Bialas et al., [3] have studied the string tension behaviour at finite temperature in three dimensional SU(3) gauge theory. Bicudo [1] compared the string tension points obtained by Bielefeld group [2] with different order parameter curves and found empirically that the ferromagnetic critical curve is the one closer to the Bielefeld data.

This work continues the study of the SU(3) string tension, first computed by the Bielefeld Group [2], and we utilize a similar technique of colour averaged free energy.

We study a wider range of temperatures in order to map the critical curve of $\sigma(T)$. In section II, we present in detail our method to extract the string tension at finite temperature. In section III we present and discuss our results and in section IV we conclude.

II. OUR SU(3) LATTICE QCD FRAMEWORK

We compute $\sigma(T)$ with in quenched SU(3) lattice QCD, fitting the linear confinement from the colour averaged free energy of a quark-antiquark pair. The colour averaged free energy of a static quark-antiquark pair is computed with the correlation of a pair of Polyakov loop and antiloop correlations,

$$\exp\left(-\frac{F_{q\bar{q}}(R, T)}{T}\right) = \langle L^\dagger(\vec{x})L(\vec{y}) \rangle, \quad (1)$$

where $R = |\vec{x} - \vec{y}|$, $T = 1/(aN_t)$ and the temperature T , in units of the Boltzmann constant $K = 1$, is the inverse of the temporal extent of the lattice,

$$T = \frac{1}{aN_t}, \quad (2)$$

a is the lattice spacing and N_t is the number of links in the temporal direction.

A Polyakov loop is a Wilson line of lattice link in the temporal direction and closed with the periodic boundary condition of the lattice,

$$L(\vec{x}) = \frac{1}{3} \text{Re Tr} \prod_{\tau=0}^{N_t-1} U_3(\vec{x}, \tau) \quad (3)$$

where $U_\mu(\vec{x}, \tau)$ with $\mu = 3$ is the time direction link. The Polyakov loop is an order parameter for the deconfinement transition, i.e. $\langle L \rangle = 0$ in the confined phase for $T < T_c$ and $\langle L \rangle > 0$ in the deconfined phase. Since a single Polyakov loop vanishes in the confined phase, to measure confinement we need a higher number of Polyakov loops. Here utilize a pair of Polyakov loop-antiloop.

The temperature and distance in Eq. (1) can be made dimensionless using the $\sigma_0 = \sigma(0)$ string tension at zero

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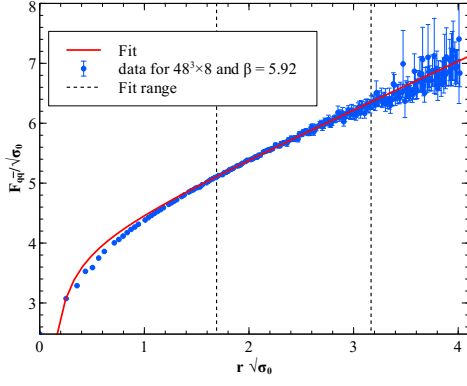


Figure 1: Colour averaged free energy results for $N_t = 8$, $L/a = 48$ and $\beta = 5.92$, at $T = 0.788T_c$. The solid line corresponds to the fit, $a_0 + a_1/r + a_2r$ with $a_1 = -\pi/12$, in the range defined by the dashed lines.

temperature ,

$$T^* = \frac{1}{N_\tau a \sqrt{\sigma_0}} = \frac{T}{\sqrt{\sigma_0}} . \quad (4)$$

and

$$r = R a \sqrt{\sigma_0} . \quad (5)$$

We fit the free energy $F_{q\bar{q}}(r, T^*)$ with

$$a_0 + \frac{a_1}{r} + a_2r, \quad (6)$$

where a_1 is fixed to the Lüscher Coulomb $\pi/12$ term and a_2 provides $\sigma(T)/\sigma_0$. We also tried a fit with a logarithmic term, however the fits were not stable, and thus we abandoned any logarithmic term.

To improve the signal in the Polyakov loop correlation functions and reduce the error, we employ the multi-hit procedure for the time links, [4, 5]. The SU(3) temporal links can be integrated out analytically and substituted by their average value,

$$U_3(\mathbf{x}) \rightarrow \bar{U}_3(\mathbf{x}) \equiv \frac{\int dU U_3(\mathbf{x}) e^{\frac{1}{3}\beta \text{Tr}(U_3(\mathbf{x})F^\dagger(\mathbf{x}))}}{\int dU e^{\frac{1}{3}\beta \text{Tr}(U_3(\mathbf{x})F^\dagger(\mathbf{x}))}} , \quad (7)$$

where $F(\mathbf{x})$ is the staple attached to a specific time link. We applied the multi-hit numerically to the time links using

$$U_3(\mathbf{x}) \rightarrow \bar{U}_3(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^n U_3^{(i)} , \quad (8)$$

with $U_3^{(i)}$ generated with the pseudo-heatbath algorithm and maintaining all the neighbouring links fixed.

To scan the temperatures, determined by Eq. (2) we utilize different β and N_t , because the lattice spacing a is a function of β . To set the scale of the lattice spacing

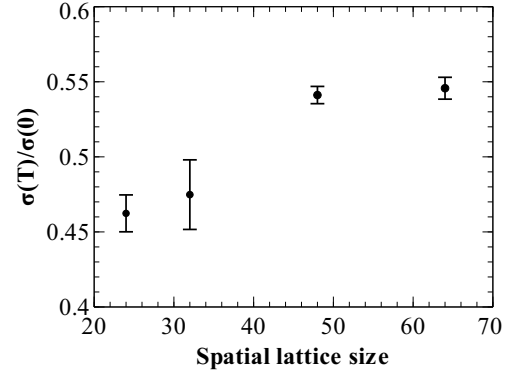


Figure 2: Large volume study of the string tension. We study σ for spatial lattice volumes $(24^3, 32^3, 48^3$ and $64^3)$ for $N_t = 4$ and $T = 0.95T_c$.

a in physical units we use the equations fitted in SU(3) lattice QCD by Edwards et al. [6],

$$(a\sqrt{\sigma_0})(g) = f(g^2) [1 + b_1 \hat{a}(g)^2 + b_2 \hat{a}(g)^4 + b_3 \hat{a}(g)^6] / b_0 \quad (9)$$

where g is the coupling constant of the Wilson action, and where Edwards et al. [6] obtained $b_0 = 0.01364$, $b_1 = 0.2731$, $b_2 = -0.01545$ and $b_3 = 0.01975$, valid in the region $5.6 \leq \beta \leq 6.5$,

$$\hat{a}(g) = \frac{f(g^2)}{f(g^2(\beta = 6.0))}, \quad (10)$$

$$f(g^2) = (b_0 g^2)^{-\frac{b_1}{2b_0}} \exp\left(-\frac{1}{2b_0 g^2}\right), \quad (11)$$

and

$$b_0 = \frac{11}{(4\pi)^2}, \quad b_1 = \frac{102}{(4\pi)^4}. \quad (12)$$

Finally, the normalized temperature, T/T_c , is given by

$$\frac{T}{T_c} = \frac{(a\sqrt{\sigma_0})(g(\beta_c))}{(a\sqrt{\sigma_0})(g(\beta))} \quad (13)$$

where β_c was obtained by Lucini et al. [7]. Thus we determine the temperature T/T_c from β and N_t .

We generate SU(3) pure gauge lattice configurations in NVIDIA GPUs, Graphical Processing Unit, (GTX295, GTX480, GTX580 and Tesla C2070). The SU(3) CUDA code uses the standard Wilson action via combination of Cabibbo-Marinari pseudoheatbath and over-relaxation algorithm by three SU(2)-subgroups, [8–10]. In order to reduce memory traffic, we store only the first two rows of SU(3) matrices in the GPU Global memory and reconstruct the third line of each matrix on the fly when

Table I: Summary of parameters and results for the string tension at finite T with $48^3 \times N_t$ lattices. For the β marked with an asterisk *, corresponding to $0.95T_c \leq T < T_c$, at finite volume, we find a contamination with some deconfined configurations. We removed the wrong configurations from our average.

β	N_t	T/T_c	$a\sqrt{\sigma_0}$	$\sigma(T)/\sigma_0$	# config.
5.69*	4	0.994	0.4011	0.3468(15)	223
5.68	4	0.971	0.4108	0.4728(77)	225
5.67	4	0.948	0.4208	0.5412(57)	450
5.66	4	0.925	0.4312	0.5879(42)	225
5.65	4	0.902	0.4421	0.6148(62)	225
5.64	4	0.879	0.4535	0.6676(179)	225
5.63	4	0.857	0.4654	0.7295(128)	225
5.62	4	0.835	0.4778	0.7439(78)	300
5.61	4	0.813	0.4908	0.7577(49)	325
5.89*	6	0.993	0.2659	0.3418(28)	206
5.87	6	0.957	0.2759	0.4408(60)	224
5.80	6	0.836	0.3159	0.7754(39)	225
5.79	6	0.819	0.3223	0.7740(64)	225
5.78	6	0.802	0.3290	0.8071(149)	225
6.062*	8	0.999	0.1987	0.4276(63)	98
6.061*	8	0.998	0.1990	0.3865(159)	93
6.06*	8	0.996	0.1993	0.3763(134)	127
6.055*	8	0.988	0.2009	0.4041(87)	175
6.05*	8	0.980	0.2025	0.4354(68)	199
6.04*	8	0.965	0.2058	0.5003(67)	238
6.03	8	0.949	0.2092	0.5336(92)	225
6.02	8	0.934	0.2126	0.5783(84)	225
6.01	8	0.919	0.2161	0.5950(47)	225
6.00	8	0.904	0.2197	0.6526(108)	228
5.99	8	0.889	0.2234	0.6974(64)	225
5.98	8	0.874	0.2272	0.7075(33)	227
5.97	8	0.859	0.2311	0.7281(205)	269
5.96	8	0.845	0.2350	0.7395(52)	225
5.94	8	0.816	0.2433	0.7683(61)	225
5.92	8	0.788	0.2520	0.7943(57)	275
5.90	8	0.760	0.2611	0.8400(130)	225
5.89	8	0.747	0.2659	0.8674(81)	227
5.87	8	0.720	0.2759	0.8853(91)	225
5.85	8	0.693	0.2865	0.9263(274)	400
5.83	8	0.667	0.2977	0.9434(136)	325
5.80	8	0.629	0.3159	0.9619(112)	402
5.72	8	0.530	0.3744	0.9966(555)	400
6.33*	12	0.989	0.1338	0.4535(153)	117
6.32*	12	0.975	0.1356	0.4473(138)	168
6.31*	12	0.962	0.1375	0.5116(214)	197
6.30*	12	0.948	0.1395	0.5163(119)	218
6.29	12	0.935	0.1415	0.5647(40)	225
6.28	12	0.922	0.1435	0.6092(53)	225
6.27	12	0.909	0.1455	0.6633(44)	225
6.26	12	0.896	0.1476	0.6437(74)	225
6.25	12	0.883	0.1497	0.7346(34)	225
6.24	12	0.871	0.1519	0.6517(172)	225
6.22	12	0.846	0.1564	0.6959(134)	223
6.20	12	0.822	0.1610	0.7311(116)	225
6.16	12	0.774	0.1709	0.8207(65)	275
6.12	12	0.729	0.1815	0.8858(33)	258
6.08	12	0.685	0.1931	0.9258(125)	325
6.00	12	0.602	0.2197	0.9750(166)	325
5.90	12	0.507	0.2611	0.9876(180)	425

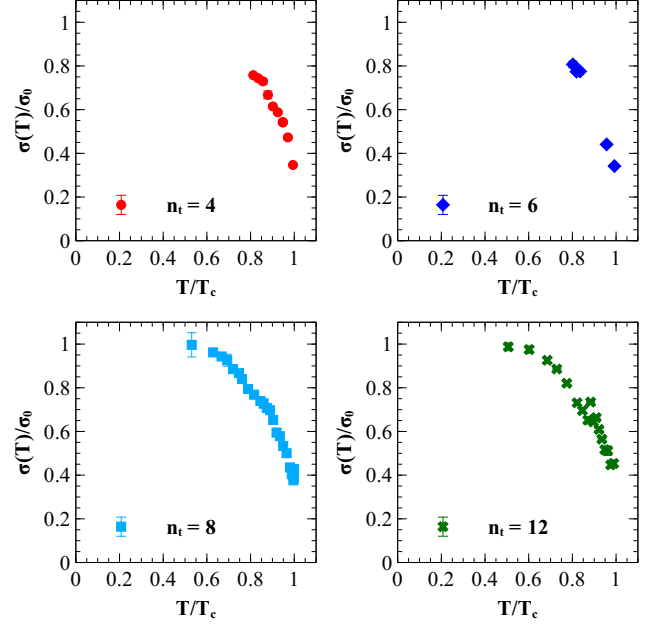


Figure 3: String tension as function of T/T_c for four temporal extensions, $N_t = 4, 6, 8, 12$.

needed. Each iteration consists of 4 pseudoheatbath and 7 over-relaxation steps followed link reunitarization. After the thermalization, in order to generate a Markov chain of configurations, we only store a configuration, to evaluate the colour averaged free energy, every 200 iterations. In order to scan a large number of temperatures, and to verify our results, we consider 57 different sets of configurations, with 225 to 450 configurations each, for different β , N_t and lattice sizes. In total we produced the large number of circa three million SU(3) configurations, thus reaching the limit of our computational facilities.

III. RESULTS FOR $\sigma(T)$ AND OUTLOOK

We fit the colour averaged free energy with Eq. (6), fixing the Coulomb term with the Lüscher term. Notice that close to T_c the linear term decreases, while the Coulomb term remains dominated by the constant Lüscher term, and we have to go to distances as large as possible in order to fit the string tension. Thus we fit the string tension in an interval at the largest distance before the statistical noise is large, and before the periodicity of the lattice saturates the free energy. The fitting range is illustrated in Fig. 1. In all our fits, the distance interval is sufficient to have a χ^2/dof smaller than one. The error in the mean average free energy, as well as the fit parameters, were determined by jackknife method.

Since large quark-antiquark distances are important, we also test the large volume limit for the configuration set with $\beta = 5.67$, $N_t = 4$ and $T = 0.948T_c$. We il-

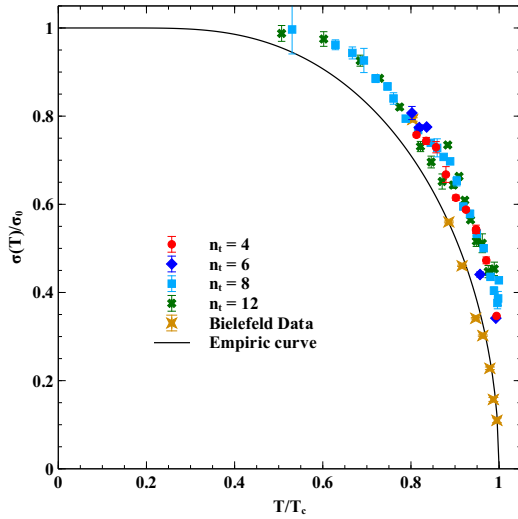


Figure 4: String tension as function of the temperature for $N_t = 4, 6, 8, 12$ combined with the results from the Bielefeld group. The black line corresponds to the ferromagnet magnetization M/M_{sat} critical curve, [1].

illustrate in Fig. 2 four different spatial lattice volumes, 24^3 , 32^3 , 48^3 and 64^3 , in order to detect spatial volume dependence effects in the results. We respectively obtain the fits for $\sigma(T)/\sigma(0)$ of 0.462 ± 0.012 , 0.475 ± 0.023 , 0.541 ± 0.006 and 0.546 ± 0.007 . For $N_t = 48$ and $N_t = 64$, the values obtained are already very close and within the statistical error. Since it would be too demanding in computer time to apply the same large volume limit extrapolation to all our lattice QCD configurations, we use the same spatial lattice volume of 48^3 in lattice units.

In Table I and in Fig. 3 we show the results of our fits for the 54 configurations sets in $48^3 \times N_t$ lattices we generate at different T . We utilize four different temporal extensions $N_t = 4, 6, 8$ and 12 because we want to scan a wide temperature range between $T = 0$ and T_c . We observe that at $T \simeq 0.5T_c$, the string tension is practically equal to the string tension at zero temperature σ_0 .

Thus, to limit the study of lattices requiring a larger N_t , we address the interval $[0.5T_c, T_c]$.

In Fig. 4, we compare the summary of our results with the results obtained by the Bielefeld group [2] and the empirical curve [1]. The Bielefeld data was obtained from lattice gauge configurations with $32^3 \times 4$ size and a tree level Symanzik-improved action. We utilize several N_t , the simpler Wilson action and a simpler fit with only a constant term, the Lüscher Coulomb term and the linear confinement. The empirical curve essentially fits most of the Bielefeld data, except for the lowest temperature one, the one at $T \simeq 0.8T_c$. Using all the configurations we generated, our points would be above the curve up to higher temperatures circa $0.95T_c$ and only beyond this temperature would get close to the empirical curve.

However, since we have a finite volume, close to T_c there is a contamination of deconfined configurations in our sets of configurations. Above $T \simeq 0.95T_c$ We observe, utilizing the history of the Polyakov loop, a pair of gaussian-like peaks [2] both in the norm and in the real part of a single Polyakov loop average. Since we have no computational power to further increase the volume, we remove the deconfined configurations from our Markov chain, with a cut between half way between the two gaussian peaks. The β where this procedure was necessary are marked with an asterisk * in Table I. Without this deconfined configurations removal, some of the string tensions $\sigma(T_c^-)$ would be as small as $0.1\sigma_0$, consistent with the results of [2]. The points below $T \simeq 0.95T_c$ together with the sets of confined only configurations at $T \geq 0.95T_c$ indicate a first order discontinuity at $\sigma(T_c^-) \simeq 0.4\sigma_0$.

To clarify the extent of the deconfinement first order transition, we plan to address, in future works, the mapping of the $\sigma(T)$ with different lattice QCD techniques.

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